What you say is what you get

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This paper is concerned with the link between teachers' talk and action in mathematics classrooms, students' talk and action in mathematics, and the mathematical meanings students make. This investigation gathered data from year 4 classrooms as teachers and students studied subtraction; using multibased arithmetic blocks, written algorithms and word problems.

In mathematics education, discourse analysis has been used to examine the way in which gender, culture and power issues interact with teachers and learners in mathematics classrooms (Ellerton & Clements, 1991; Mousley & Marks, 1991; Walkerdine, 1988; Zevenbergen, 1993). But there appear to be few studies investigating the connection between the detail of what teachers say and do, and what students actually learn. This research investigated the impact of teachers' language and actions on students' language and actions, as teachers taught subtraction algorithms to Year 4 students.

Method

Five teachers used two teaching approaches in teaching six Year 4 classes subtraction algorithms. One teaching approach emphasised mathematical relationships as Multibased Arithmetic Blocks (MABs) were used to develop and solve subtraction algorithms. Mathematical relationships were emphasised with the materials, within the written algorithms, and between these representation systems. In particular, reasons were always sought from students as to what the materials and actions on them represented, what the written algorithm represented, and what steps followed from a given situation and why. The second approach used MABs to complete subtraction questions, and to establish a written algorithm, but the detail of the mathematical relationships were not emphasised to the same extent.

Thirty-two lessons were videotaped, and 24 students were interviewed at the completion of teaching, and again four weeks later. During interviews, students were videotaped as they completed subtraction problems, and as they explained their actions and reasoning. All videotapes were transcribed, and the transcriptions modified, so as to provide a record of actions on teaching materials.

Data and analysis

Teaching approach 1

In the following lesson, the teacher draws a place value chart on the board, she writes 231 on the board in place value positions, and asks students to show the number 231 with MABs. She checks that students have the correct materials, both by asking them and by looking about the room, then explains she is going to do something different, and writes *-40* on the chart. The teacher and several students establish that, in this case, renaming will occur in the tens column, not in the units column.

T: So we have been told by some very clever people that we are not renaming in the units column. So will you tell me what does it say here? (points to units column, 1-0)

S: One take away nought.

T: Is?

S: One (teacher writes 1 in answer space).

T: Now what do you say in this column (points to tens column, 3-4)? You tell me?

S: Three take away four, you can't do it.

T: So we have 3 tens, take away 4 tens, we can't do that. What do I do?

S: You trade the um ....

T: You're right, you trade. What are you trading?

S: Two

T: Can someone help him?

S: You cross off the three (corrects himself) I mean you cross off the two and put a .... just one (teacher does this).

T: That's one what?

S: One hundred.

T: (disciplinary remark)

S: And you put the one next to the three (teacher does this).

T: What's this now?

S: Thirteen.

T: Thirteen what?

S: Take away..

T: No. Thirteen what? Thirteen units?

S: Thirteen tens.

T: Thirteen tens. Now, the only thing that's different is that we've moved over into the tens column, the same pattern (teacher at board using both hands to show how pattern for 10s/1s is moved over to the 100s/10s). We do the same thing we did before. Now ....

In this lesson the teacher insisted on particular actions with MABs, and in the way the algorithm was written down. She checked students' work as she moved about the class. Her questions directed the actions and writing patterns she wanted, but also emphasised the relationships within the blocks, and between the blocks and the written algorithm. This was also the case with other teachers using this teaching approach as illustrated in the next text.

T: Okay let's look at the tens column. We have three tens and we are asked to take away how many?

S: Eight.

T: Can you take eight from three?

S: No.

T: Okay, we can't, so what are we going to do? We are going to?

S: Trade.

T: We are going to trade a hundred for how many tens?

S: Ten.

T: Ten tens. We have ten tens and three tens, we join them together and we have how many?

S: Thirteen.

T: Thirteen. Don't forget to write it in.

Here the teacher insisted that children watch and follow her instructions, she asked questions continually, to gauge if children understood: and she emphasised place value, decomposition and renaming at every step. She emphasised both written and MAB procedures, and described the relationship between the written algorithm and actions on the MABs.

These texts indicate that in the first teaching approach, teachers emphasise every aspect of the use of MABs and the written algorithm. Through explanation, demonstration and questioning, they show relationships within the MABs, within the algorithm and between actions on the MABs and the written algorithm. Liza had participated in this teaching approach: here she is calculating 547-169, as a written algorithm.

R:	Can you tell me why that one is there?	The 1 above the 7
S:	Because that one ten was traded and so you could put it here.	*Logical relationship*
R:	So you had one ten and it was traded and ten came there and that number became what?	Fact and relationship.
S:	Seventeen units.	
R:	Seventeen units, good girl. Now you crossed that four out and you wrote something up here. Now why did you cross the four out and write something at the top?	*Relationship*, and *technical* terms."*I*" implies identification with action and as agent.
S:	Because I needed to trade so I traded one of the tens for ten units and then I only had three tens left.	*Recognises* the reason to trade, uses "*I*".
R:	Good girl, that's very good, you had four tens and you needed to trade one so you wrote the three up there and one went over here. Now what about here, why did you have to cross that five out?	
S:	Because I couldn't take six from three so I had to trade one of the five tens for a four *(pause)* I mean five hundreds for a four and then I put one of the ten units in that one so you could take six from thirteen.	*Corrects* herself. Full description. Uses *technical* terms.

This student had a good understanding of the procedure required to complete the algorithm, used correct terminology, identified facts and logical relationships in the algorithm, corrected herself when she made an error, and gave full descriptions of sections of the algorithm without requiring further prompting. Later in the interview, when she used MABs to answer 653-472, Liza traded correctly, and was able to describe the logical relationship inherent in her actions: *we take two from three and we have one left, put that down there and then we have seven from five so we can't do it and we put that back. Then you take seven*. As she continued with the MABs, she was able to explain her actions: *Since you can't take seven from five you trade one of these.*

Liza uses materials in an effective manner, leading to a correct solution. But there is much more than this happening. The student recognises and applies the relationship between the actions on the materials and the written algorithm. Her descriptions and explanations contain many logical relationships, she uses *then, so* and *since*, to describe her logic. Indeed her answers go beyond what one might expect. For example, when asked to say what happens now, she not only describes the next step, but also explains why. She identifies meanings and establishes relationships within the algorithm, and between the algorithm and the teaching aid: and explains using a logical relationship rather than by referring to a rule. Her understanding seems to go beyond the procedural, to one involving an understanding of relationships.

Teaching approach 2

The text below is taken from the second teaching approach. The teacher had children work in parallel with her: sitting in a group, on the floor, at the front of the room. Here they complete 42 - 7, using MABs.

T: Oh, 42 is it? One, two, three, 42. Ten, 20, 30, 40. Two. I'm working upside down. What do I do now?	Teacher counts out 4 longs, starts counting 1,2,3, corrects herself, 10,20,30,40. Then places 2 units.
S4: Take the ten away and you get ten ones.	
T: Why?	
S4: You can't trade.	
T: What do I need to trade? S: Because you can't take away seven. T: Oh. Excellent boy. One, two, three, four, stop fiddling, ... ten. I've now traded, two, four, six, eight, ten.	Teacher holds a ten in her left hand, takes 10 units from the bank with her right hand, places them next to the tens, counts as she does it, then puts the ten in the bank.
What do I do now?	
S5: Take away seven from the ten T: Exactly. One, two, three, four, five, six, seven. What's my remainder?	Teacher takes seven units away from the line of ten
S: Thirty-five. T: 10, 20, 30, one, two, three, four, five.	Teacher counts tens and units, verifies answer.

The point I want to make here, is that the teacher subtracted from the traded 10 units. That is, she did not emphasise joining the units together and renaming them. This is an acceptable part of this second teaching approach, and one I would claim as common in classrooms. What then do students do when they are asked to complete subtractions using MABs? The extent to which they copy the teacher's actions are shown in the samples below.

Student 1 91-38 = 9 tens - 38
= 80+10-38 trades one 10 for ten 1s
= 80+10+1-38 realises needs 1 in units place
= 90+10+1-38 (adds one ten), long pause
= 90+(11-8)-30 (subtracts 8 from 1 unit and traded 10)
= 80+3+10-30 (trades 1 ten for 10 units)
= 90+3-30 (undoes trade, subtracts three 10s)
= 60+3
= 63 (incorrect)

In the text above, Student 1 starts with 9 tens, and trades for 10 units, then realises the need for another unit but keeps it separate from the traded units. The student adds another ten, and after a time combines the units and subtracts 8 from 11. The error involves the additional ten taken from the bank. The problems here are related to poor representation of numbers using MABs, and an inadequate proceduralisation of when to trade and the trading process. In the second example below, the student correctly represent 75 with MABs, then trades 1 ten for 10 units.

Student 2 75-46 = 60+10+5-46 (three piles, six 10s, ten 1s, five 1s)
= 60+(10-6)+5-40 (subtracts 6 from 10)
= 60+4+5-40 (subtracts four 10s, units not combined)
= 20+4+5 (long pause, counts ten 1s from bank, holds them for some seconds, returns them to bank)
= 20+9 (combines 1s)
= 29 (correct)

After trading, this student did not join the units together, and subtracted 6 from 10. The units were still not combined when the student subtracted 4 tens from 6 tens. The units are then combined and the student achieved the correct answer. The point here is that the answer was correct, but the manipulation of MABs did not help to establish an efficient written algorithm. That is, the level of correspondence between actions on blocks and the written algorithm was low. Further, this subtraction from the traded 10 units was common, with the teacher's procedure copied as it was presented in the earlier text, even though she made no special mention of it. Here are more samples of students' actions on MABs from this second teaching approach.

83-59	72-39	134-27
= 70+10+3-59	= 60+10+2-39	= 100+30+4-27
= 70+(10-9)+3-50	= 60+(10-9)+2-30	= 100+20+(4-4)+(10-3)-20
= 70+1+3-50	= 60+1+2-30	= 100+20+7-20
= 70+4-50	= 60+3-30	= 100+7 (correct)
= 20+4	= 30+3	= 1 7 (Writes 1_7, space between digits)
= 24 (Writes 24, correct)	= 33 (Writes 33, correct)	

In each case these students obtained the correct answer, but equally in each case, their methods involved actions for which there can be no correspondence in an efficient written algorithm. The algorithms I have written reflect their actions: to the extent that this also reflects their cognitive processing, they have an inefficient cognitive network. In these cases it seems MABs are being used as calculators, not as a vehicle for furthering understandings.

Discussion

My argument here then, is that the approaches used by students typically reflect the words and actions of the teacher. Indeed, students' actions will reflect both what the teacher intends and what may not be intended. Teachers in the second teaching approach did not emphasise joining and renaming, so some students joined traded units with existing units, others didn't. Some students subtracted from the joined units, some subtracted from the traded tens, some found other possibilities. This has implications for the development of effective teaching approaches, especially where the context requires algorithmic procedures.

When Steve, a student in the second teaching approach, used MABs to calculate 83-37, he first used MABs to represent 83, then subtracted three tens from the eight tens as his first action after forming 83. This may be arithmetically correct, but it does not help establish the procedure for completing a written algorithm. His lack of a deeper understanding of the materials, of his actions on them and of the various interrelationships that exist, is reflected in his explanation: *Because I do it*. That is, he could explain his actions only in procedural terms, and was unable to answer a series of *why* questions. He referred to the trade action as *took one of them away*, as he placed a ten in the bank. In the case of subtracting seven, he says *seven*, then brings a unit from the bank for each of *eight, nine, ten*. That is, he calculates 10-7, then adds the existing 3 units to give the answer 6. Again these methods reflect an idiosyncratic procedure, which may arrive at the correct answer, but one not reflecting either the way the student writes the algorithm, or the way a written algorithm is efficiently completed.

Students in this second teaching approach, more frequently than their counterparts in the first teaching approach, were unable to regularly and efficiently obtain correct answers to subtraction algorithms, or to provide explanations for their actions that made reference to logical relationships or to mathematical constructs. Their use of MAB materials suggested a lack of understanding about the relationships within the materials and actions upon them, and insufficient insight into the way the materials related to written algorithms. That is, their words and actions reflect the teaching approach where the relationships within and between representation systems are either implied or discussed only briefly.

That is, I am arguing that there is a link between teaching approach and the way students complete algorithms, the way they use and make sense of concrete materials, and the manner in which they speak and think about subtraction. Unless opportunity exists for students to make explicit the relationships within and between representation systems, then for a good many students these relationships will not be established. In this context, teaching approach 1 does appear to help students establish relationships more effectively than teaching approach 2.

I am not arguing that teaching approach 1 is the only method likely to be successful: there may be a range of other successful approaches. But the learning outcomes of teaching approach 2 indicate that if relationships and procedures are not emphasised then learners will invent their own. This may be desirable, but too informal an approach is unlikely to have the desired learning outcomes.

References

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